Homework 6

Total 20 points

Due November 26th at 11:59pm

Problem 1. (5 points) If function f is μ-strongly convex, and $\mathbf{g}$ is a subgradient of f at $\mathbf{x}$. Show that for any $\mathbf{y} \in dom\ f$,

$$f(\mathbf{y}) \geq f(\mathbf{x}) + \langle \mathbf{g}, \mathbf{y} - \mathbf{x} \rangle + \frac{\mu}{2} \|\mathbf{y} - \mathbf{x}\|_2^2 .$$

Problem 2. (5 points) Suppose f is convex and G-Lipschitz continuous over the constraint $\mathcal{C}$, which is bounded and convex with diameter $D > 0$. If we run projected subgradient descent method for T rounds with $\eta_t = \frac{D}{G\sqrt{T}}$, then we have

$$f(\bar{\mathbf{x}}_t) - f^* \leq \frac{DG}{\sqrt{T}},$$

where $\mathbf{x}_t = \frac{1}{T} \sum_{t=1}^{T} \mathbf{x}_t$.

Problem 3. (10 points) Let f be μ-strongly convex and G-Lipschitz continuous over the constraint $\mathcal{C}$. Let $\eta_t = \frac{2}{\mu(t+1)}$ and $\bar{\mathbf{x}}_t = \sum_{k=1}^{t} \frac{2k}{t(t+1)} \mathbf{x}_k$. Prove that the projected subgradient descent obeys

(a)

$$f(\bar{\mathbf{x}}_t) - f^* \leq \frac{2G^2}{\mu(t+1)};$$

(b)

$$\|\bar{\mathbf{x}}_t - \mathbf{x}^*\|_2 \leq \frac{2G}{\mu\sqrt{t+1}}.$$