Homework 5

Total 20 points

Due November 19th at 11:59pm

Problem 1. (5 points) Compute the projection $\mathcal{P}_{\mathcal{C}}(\mathbf{x})$ for the following sets:

(a) (2 points) halfspace: $\mathcal{C} = \{\mathbf{x} | \mathbf{a}^\top \mathbf{x} \leq b\} (\mathbf{a} \neq 0)$;

(b) (3 points) unit ℓ_1 ball: $\mathcal{C} = \{\mathbf{x} | \|\mathbf{x}\|_1 \leq 1\}$.

Problem 2. (5 points) Suppose f is a convex and differentiable function, $\mathcal{C}$ is a closed convex set. Show that

$$\mathbf{x}^* \in \arg\min_{\mathbf{x} \in \mathcal{C}} f(\mathbf{x}) \iff \langle -\nabla f(\mathbf{x}^*), \mathbf{z} - \mathbf{x}^* \rangle \leq 0, \ \forall \ \mathbf{z} \in \mathcal{C}.$$

Problem 3. (5 points) Consider the projected gradient descent algorithm introduced in the class. Suppose that for some iteration t, $\mathbf{x}_{t+1} = \mathbf{x}_t$. Prove that in this case, $\mathbf{x}_t$ is a minimizer of the convex objective function f over the closed and convex set $\mathcal{C}$.

Problem 4. (5 points) Let $\mathcal{C} \in \mathbb{R}^d$ be a nonempty closed and convex set, and let f be a strongly convex function over $\mathcal{C}$. Prove that f has a unique minimizer $\mathbf{x}^*$ over $\mathcal{C}$.