Homework 2

Total 30 points

Due October 29th at 11:59pm

Problem 1. (6 points) Judge which of the following functions are (strict) convex.

(a) $f(x_1, x_2) = x_1 x_2$, $x_1 > 0, x_2 > 0$.

(b) $f(x_1, x_2) = 1/(x_1 x_2)$, $x_1 > 0, x_2 > 0$.

(c) $f(x_1, x_2) = x_1^2/x_2$, $x_2 > 0$.

Problem 2. (6 points) Prove that $f : \mathbb{R}^n \to \mathbb{R}$ is convex if and only if for all $\mathbf{x}, \mathbf{y} \in \text{dom } f$ and $\mathbf{x} \neq \mathbf{y}$, the function $g(t) = f(t\mathbf{x} + (1-t)\mathbf{y})$ is a convex function on $[0, 1]$.

Problem 3. (6 points) Prove that for any convex function $f : \mathbb{R}^n \to \mathbb{R}$, the Bregman distance $B_f(\mathbf{x}, \mathbf{y}) = f(\mathbf{x}) - f(\mathbf{y}) - \langle \nabla f(\mathbf{y}), \mathbf{x} - \mathbf{y} \rangle$ is convex in $\mathbf{x}$ but not necessarily in $\mathbf{y}$.

Problem 4. (6 points) Compute the subdifferentials of the following functions

(a) $f(\mathbf{x}) = \|\mathbf{x}\|_2$.

(b) Given a closed convex set $\mathcal{C}$, define

$$f(\mathbf{x}) = \begin{cases} 0 & \text{if } \mathbf{x} \in \mathcal{C} \\ +\infty & \text{otherwise.} \end{cases}$$

Problem 5. (6 points) If function f is convex, Show that $\partial f(\mathbf{x}) \neq \emptyset$ for all $\mathbf{x} \in (\mathit{dom}\ f)^\circ$.