# Notes for Lecture 1

Scribe: Cheng Chen

## 1 Some Properties of Norms

We provide some properties of norms that wasn't covered in the class. You can verify these properties by yourself. Notice that in our class, we only study the properties of real matrices.

**Property 1.** *For and matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ and orthogonal matrix $\mathbf{Q} \in \mathbb{R}^{m \times m}$, we have*

$$\|\mathbf{Q}\mathbf{A}\|_2 = \|\mathbf{A}\|_2, \qquad \|\mathbf{Q}\mathbf{A}\|_F = \|\mathbf{A}\|_F.$$

**Property 2.** *For and matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$, we have*

$$\|\mathbf{A}\|_F = \sqrt{\operatorname{tr}(\mathbf{A}^\top \mathbf{A})} = \sqrt{\operatorname{tr}(\mathbf{A}\mathbf{A}^\top)}$$

## 2 Some Properties of SVD

**Property 3.** *Suppose the rank of $\mathbf{A}$ is $r$, then*

$$\|\mathbf{A}\|_2 = \sigma_1, \qquad \|\mathbf{A}\|_F = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_r^2}.$$

**Property 4.** *The nonzero singular values of $\mathbf{A}$ are the square roots of the nonzero eigenvalues of $\mathbf{A}^\top \mathbf{A}$ or $\mathbf{A}\mathbf{A}^\top$.*

**Property 5.** *If $A = A^\top$, then the singular values of $\mathbf{A}$ are the absolute values of the eigenvalues of $\mathbf{A}$.*