Homework 5

Deadline: November 6th.

Problem 1. Suppose f is a convex and differentiable function, $\mathcal{C}$ is a closed convex set. Show that

$$\mathbf{x}^* \in \arg\min_{\mathbf{x}\in\mathcal{C}} f(\mathbf{x}) \iff \langle -\nabla f(\mathbf{x}^*), \mathbf{z} - \mathbf{x}^* \rangle \leq 0, \ \forall\, \mathbf{z} \in \mathcal{C}.$$

Problem 2. Consider the projected gradient descent algorithm introduced in the class. Suppose that for some iteration t, $\mathbf{x}_{t+1} = \mathbf{x}_t$. Prove that in this case, $\mathbf{x}_t$ is a minimizer of the convex objective function f over the closed and convex set $\mathcal{C}$.

Problem 3. Let $\mathcal{C} \in \mathbb{R}^d$ be a nonempty closed and convex set, and let f be a strongly convex function over $\mathcal{C}$. Prove that f has a unique minimizer $\mathbf{x}^*$ over $\mathcal{C}$.